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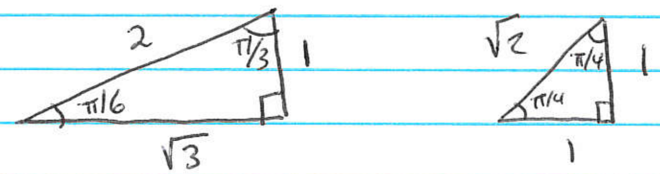
① $2\ln(x) - \ln(y) + 1 = \ln(x^2) - \ln(y) + \ln(e)$

$$= \boxed{\ln\left(\frac{ex^2}{y}\right)}$$

② $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{-\pi}{4}\right) + \sin\left(\frac{-\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)$



$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

③ ⓐ $102\,m - (5\,m/s^2)t^2 = 2m$

$$(5\,m/s^2)t^2 = 100\,m$$

$$t^2 = 20\,s^2 \Rightarrow \boxed{t = \sqrt{20}\,s}$$

-or- $\boxed{t = 2\sqrt{5}\,s}$

ⓑ $v(t) = h'(t) = -2(5\,m/s^2)t$

$$v(\sqrt{20}\,s) = -2(5\,m/s^2)(\sqrt{20}\,s) = -10\sqrt{20}\,m/s$$

$$= \boxed{-20\sqrt{5}\,m/s}$$

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(7) (a) $\lim_{x\to 0} x \csc(x) = \lim_{x\to 0} \frac{x}{\sin(x)} = \boxed{1}$

(b) $\lim_{x\to\infty} x - \sqrt{x^2+1} \overset{*}{=} \lim_{x\to\infty} x\left[1 - \sqrt{1+1/x^2}\right]$

$= \lim_{x\to\infty} \frac{\overset{\to 0}{1-\sqrt{1+\frac{1}{x^2}}}}{\underset{\to 0}{1/x}} \overset{\text{l'Hôp}}{=} \lim_{x\to\infty} \frac{\frac{1}{\cancel{2}\sqrt{1+1/x^2}}\left(\frac{\cancel{+2}}{x^3}\right)}{\left(\cancel{-}1/x^2\right)}$

$= \lim_{x\to\infty} \frac{\cancel{x^2}}{\sqrt{1+1/x^2}\ x^{\cancel{3}}} = \lim_{x\to\infty} \frac{1}{x\sqrt{1+1/x^2}} = \boxed{0}$

(c) $\lim_{\theta\to \pi/2^-} \tan(\theta) - \sec\theta = \lim_{\theta\to\pi/2^-} \frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}$

$= \lim_{\theta\to\pi/2^-} \frac{\overset{\to 0}{\sin\theta - 1}}{\underset{\to 0}{\cos\theta}} \overset{\text{l'Hôp}}{=} \lim_{\theta\to\pi/2^-} \frac{\cos\theta}{-\sin\theta} = \frac{\cos(\pi/2)}{-\sin(\pi/2)} = \boxed{0}$